Quaternary Research xxx (2013) xxx–xxx

Contents lists available at SciVerse ScienceDirect

Quaternary Research

journal homepage: www.elsevier.com/locate/yqres

Are drumlins a product of a thermo-mechanical instability?

Roger LeB. Hooke *, Aaron Medford

School of Earth and Climate Sciences and Climate Change Institute, University of Maine, Orono, ME 04469-5790, USA

ARTICLE INFO

Article history:
Received 5 June 2012
Available online xxxx

Keywords:
Glacial geology
Drumlins
Glacial landforms
Instability

ABSTRACT

Of numerous theories of drumlin genesis, none has been widely accepted. It seems evident, however, that some form of positive feedback process is involved. Under certain circumstances perturbations are amplified. Herein we suggest that patchy areas of frozen bed provide the initial perturbation. Such frozen patches may occur in local areas underlain by material of lower thermal conductivity or on slight topographic highs. Drag exerted by the frozen patch deflects ice flow into its lee, dragging with it mobile till eroded from the thawed area. The energy balance is such that this till likely refreezes, either producing a topographic perturbation or amplifying an existing one. The resulting topography then deflects more of the geothermal heat away from the developing hill and into the adjacent trough, resulting in a positive feedback. Once the thermal perturbation exceeds a critical (though as yet undefined) level, melting may decouple the ice from the bed, preventing further entrainment of till from thawed areas, and thus limiting the height and length of the drumlin.

Introduction

Previous work

Glaciers are impressive earth-moving machines so it is hardly surprising that, under suitable conditions, they should produce streamlined forms. Examples include crag-and-tail features on bedrock outcrops, roche moutonnées, rock drumlins, drumlins, flutes, and mega-scale glacier lineations. Herein, our focus is on drumlins. Drumlins are elongate streamlined hills with modest aspect ratios, horizontal dimensions of order 10^2 to 10^3 m and heights of order 10^0 to 10^1 m. They are common features of the beds of former ice sheets (e.g. Colgan and Mickelson, 1997; Hess and Briner, 2009), and have also been described from the margins of contemporary ice caps (Hart, 1995; Johnson et al., 2010). They likely form in several ways. At one end of the spectrum are those that may have arisen spontaneously from flow over a relatively flat homogeneous bed; at the other end are those formed by simple streamlining of preexisting hills. As Clark (2010) noted, many models require a process that amplifies the relief. Rates of formation must also vary. Smith et al. (2007) appear to have documented formation of one beneath an Antarctic ice stream within 7 yr.

Our interest is in drumlins formed under glaciological conditions that are tamer than those beneath ice streams. These drumlins commonly occur in fields of several tens or hundreds. We speculate that they can arise from flow over a relatively flat bed, but not a homogeneous one. The inhomogeneities, we suggest, are materials that are more resistant to deformation by the overriding ice, a possibility first proposed by Smalley and Unwin (1968).

Smalley and Unwin (1968) thought that before an ice sheet could erode a bed consisting of clastic material (e.g. till), the material would have to dilate. They reasoned that patches of coarser till would resist dilation. Erosion on either side of these patches (and likely deposition on them) would then amplify the perturbation. Baranowski (1977) suggested that the resistant patches, rather than simply being coarser, might be frozen. He viewed these frozen patches as occurring at a phase-change boundary from a thawed to a frozen bed, and believed that the patches might develop when atmospheric temperatures periodically dropped for short periods of time immediately preceding and during deglaciation.

More recently, Hindmarsh (1998, 1999), Fowler (2000, 2009, 2010), Schoof (2007), and Chapwanya et al. (2011) have explored the possibility that drumlins form from an instability arising from ice flow over deformable till. Instabilities are situations in which an *infinitesimal* perturbation sets up positive feedbacks that amplify the perturbation. Fowler (2009) compared the drumlin-forming process with the way water flowing over sand generates ripples and dunes. Hindmarsh (1999) and Fowler (2010) used a linearly viscous ice rheology in their calculations and were able to show that certain combinations of till thickness and rheology, sliding speed, and perturbation wavelength led to development of waves in the ice–till interface *normal to the direction of ice flow*. They thought ribbed moraine might be a result of this process.

Schoof (2007, p. 228) paraphrases the Hindmarsh-Fowler instability mechanism as follows: "When ice flows over a shallow bump in the ice–till interface, it exerts higher compressive normal stresses on the upstream side of the bump than on its downstream side. If the viscosity of ice is…much greater than that of till…more sediment flows [longitudinally] into the bump than out of it. [T]his causes the bump to grow."

Schoof extended the Hindmarsh-Fowler analysis to more plastic till rheologies, but was still unable to identify a three-dimensional instability

* Corresponding author. Fax: +1 207 581 2202.
E-mail address: rogerhooke@gmail.com (R.L. Hooke).

0033-5894/$ – see front matter © 2012 University of Washington. Published by Elsevier Inc. All rights reserved.
http://dx.doi.org/10.1016/j.yqres.2012.12.002

that might lead to drumlins. He thought the model was likely conceptually flawed as it predicted migration of the drumlins downflow, whereas the cores of some drumlins are composed of fluvial gravels that were unlikely to have migrated. These gravels were probably deposited in a proglacial environment prior to advance of the ice. Fowler (2009), however, modified Schoof's approach and obtained waves that were stationary but still two-dimensional.

Drumlin characteristics and likely conditions of formation

In a comprehensive review of literature on 95 drumlin fields worldwide, Patterson and Hooke (1995) found that fields of distinct well-developed drumlins tended to occur within ~80 km of an associated ice margin, but separated from it by a drumlin-free zone. This is illustrated particularly well by drumlin fields left by the Green Bay lobe (Wisconsin, USA) as it retreated. The axes of drumlins in each field are normal to the recessional moraine behind which they are believed to have formed, but are not normal to earlier recessional moraines further away (Colgan and Mickelson, 1997). Some authors (e.g. Clark, 1999 and written comm., July 2012) think drumlins can form much further from the margin, but firm evidence for this appears to be lacking. This is certainly, in part, because it becomes increasingly difficult to reliably chronologically correlate a drumlin field with a margin as the distance between the two increases.

The area within several tens of kilometers of an ice sheet's margin is one (the ablation zone) in which longitudinal strain rates were likely becoming compressive, vertical velocities upward, and the ice relatively thin. Down-ice-diverging drumlin axes suggest that transverse strain rates were commonly extending. Numerical modeling (Patterson and Hooke), till deformation (Stanford and Mickelson, 1985; Menzies et al., 1997), rare fluvial interbeds (Goldstein, 1994), and other lines of evidence suggest that the basal temperature was at the pressure melting point when drumlins formed. Patterson and Hooke also noted that many well-developed drumlin fields lie upglacier from an ice margin that either ended in the sea or showed evidence of being frozen to the bed. This led them to argue that the pore water pressure was likely high, promoting bed deformation.

Patterson and Hooke found that drumlin shape and composition, substrate material and thickness, and the broad-scale topography of drumlin fields were highly variable, suggesting that none of these are crucial for drumlin formation. Stokes et al. (2011) reached the same conclusion, and suggested that a single process (or suite of processes) were likely responsible for the continuum of drumlin forms. They concluded that any proposed mechanism of drumlin formation must be able to account for the wide range of morphologies seen. The intensive search for this mechanism, starting in the late 19th century (e.g. Tarr, 1894 and references therein) serves as a reminder of our lack of understanding of a common subglacial process.

Basal thermal conditions

Parts of the beds of the Greenland and Antarctic ice sheets are at the pressure-melting temperature and others are frozen (Oswald and Gogineni, 2008; Bell et al., 2011). The same was undoubtedly true of the now-vanished ice sheets of the Pleistocene (e.g. Kleman and Glasser, 2007). A likely location for a transition from a frozen bed (up-ice) to a thawed one (down-ice) is beneath the equilibrium line. Near the equilibrium line, vertical ice velocities change from downward in the accumulation zone to upward in the ablation zone. This reduces the thermal gradient in the ice, thus “trapping” geothermal and frictional heat at the bed (Hooke, 1977). Intuitively, such transitions must occur over distances of several kilometers, with patches of frozen bed surrounding or surrounded by thawed areas (Fig. 1; Hughes, 1992). This patchiness could be associated with slight topographic irregularities (e.g. Kleman and Borgström, 1994) or with inhomogeneities in thermal conductivity of the bed material. Where conditions are right (and we do not know exactly what ‘right’ is in this context), erosion is likely in the thawed areas, resulting in development of some relief (or amplification of relief already present).

The idea that frozen areas might stimulate drumlin growth is over 60 yr old; Armstrong and Tipper (1948, p. 293) suggested that ‘a knob of frozen till’ might do the trick. Other glacial landforms have also been attributed to processes in the vicinity of a water/ice phase change boundary at the bed. Most notably: (*i*) Hättestrand and Kleman (1999) have attributed some ribbed moraine to tensional fracturing of a meters-thick layer of frozen till, (*ii*) Kleman and Borgström (1994) describe landforms developed around frozen patches in areas of likely *extensional* ice flow, and (*iii*) Jansson and Kleman (1999) describe bedrock knobs with paired horns of till extending down-ice. Jansson and Kleman attribute these latter landforms to flow on a thawed bed at lower elevations around a hilltop that is frozen. Preservation of the landform, they posit, requires that totally frozen bed conditions resume later.

Our conceptual model

If a frozen patch is strong enough to resist entrainment by the glacier, it increases drag on the glacier sole relative to that in adjacent thawed zones. Conservation of mass then requires that a slight dimple develop in the glacier surface. (Depressions in the lees nunataks are an extreme form of this dimpling.) Surface slopes into the dimple from either side drive a herringbone flow at the bed in the lee of the up-ice edge of the frozen patch (Fig. 2). Such flow is also theoretically possible in a purely 2-D situation, as on the flanks of a flute of uniform height (Schoof and Clarke, 2008). Evidence for such flow is seen in the herringbone patterns found in till fabric studies on some drumlins (Andrews and King, 1968; Embleton and King, 1975, p. 412) [although not on others (e.g. Walker, 1973)] and on flutes (Shaw and Freschauf, 1973; Rose, 1989; Benn, 1994). [Most till fabric analyses on drumlins are done in places, as along the drumlin axis or at the stoss end, where one would not expect to see the herringbone pattern. Others are at cross sections exposed by erosion an unknown distance from the stoss end (e.g. Andrews and King, 1968; Embleton and King, 1975). As a herringbone pattern is expected to be best develop midway along the drumlin (Fig. 2), these studies are inconclusive.] Under suitable conditions, the ice sliding diagonally into the lee of such a perturbation will drag wet till with it.

Both topographic and thermal inhomogeneities affect heat flow to Earth's surface. Isotherms deep in the earth are roughly spherical, parallel to Earth's mean surface. Just beneath the surface, however, isotherms must warp to conform to the topography (Fig. 3) or to the presence of a cold patch. Heat flow is normal to isotherms, so near the surface beneath a topographic irregularity geothermal heat moves towards valleys and away from ridges (Fig. 3; Lees, 1910; Lachenbruch, 1968; Turcotte and Schubert, 2002). A frozen patch resulting from a conductivity contrast would have the same effect, albeit with a more complicated temperature distribution than that shown in Figure 3.

(For convenience in the following discussion, we refer only to topographic perturbations, but the principles apply equally well to perturbations resulting from conductivity contrasts. We also refer to the “ridge” and “trough” of the perturbation. These regions are identified in Figure 3. The ridge is assumed to be frozen and the trough thawed.)

In the ice immediately above the bed, the curvature of the heat flux lines is reversed. Thus, more heat is conducted upward into the ice from ridges than from troughs (Fig. 3).

Thus, the energy balance in the troughs is positive (more heat reaching the interface from below than is withdrawn upward), leading to melting. Conversely, the energy balance over ridges is negative, resulting in freezing.

The negative heat balance on the ridges should freeze at least some of any till dragged to this location by the converging ice flow, increasing the height of the perturbation. At the same time, the trough is being deepened by erosion. This increase in relief increases the difference in heat flux, and growth of the perturbation proceeds in a positive

Please cite this article as: Hooke, R.L., Medford, A., Are drumlins a product of a thermo-mechanical instability?, Quaternary Research (2013), http://dx.doi.org/10.1016/j.yqres.2012.12.002

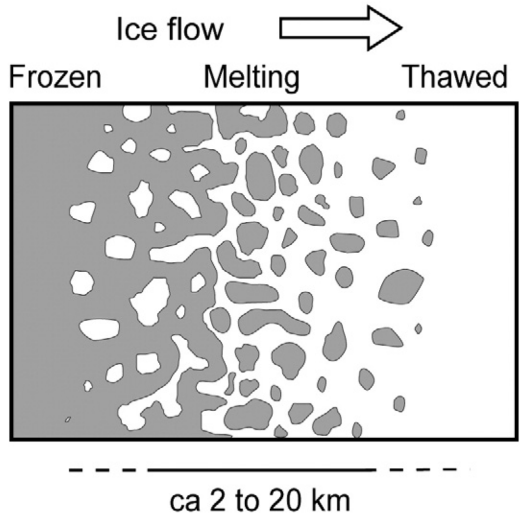

Figure. 1. Hypothetical distribution of frozen and thawed areas within a transition zone from a region of frozen bed to one of thawed bed (modified from Hughes, 1992, Fig. 14).

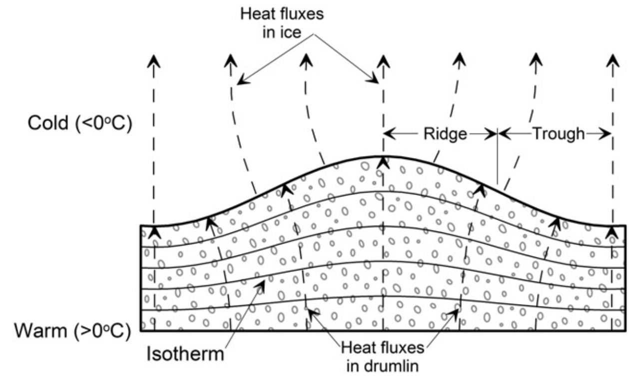

Figure. 3. Schematic sketch of heat flux beneath and above an undulating ice/bed interface. Thin solid lines are isotherms. Dashed lines show directions of heat flow (normal to isotherms). The heat moving between any two adjacent flux lines is equal everywhere in the sketch. Thus, where flux lines are closer together, the *flux per unit area* is higher, and conversely. Therefore, more heat per unit area reaches the trough than the crest, and more is conducted away from the crest.

feedback process. Later, we speculate on a possible process limiting growth, and also one that might lead to the down-glacier increase in drumlin elongation.

We emphasize that, at this stage, our model applies only to elongate features with comparatively low aspect ratios (drumlins) and to those formed where ice was flowing as a sheet (not as an ice stream). We doubt that it is applicable to mega-scale glacial lineations or to small-scale flutes. It is also more likely to be applicable to drumlins in fields, rather than to isolated forms that could well be a result of streamlining of pre-existing hills.

Heat-flux calculations for a topographic perturbation

Various approaches have been used to calculate heat-flux variations due to topography. We have modified one by Turcotte and Schubert (2002, p. 147–149), originally developed to illustrate the need for topographic corrections to borehole temperature measurements in order to calculate geothermal fluxes. Our perturbation is in the form of a sinusoidal surface with a wavelength, λ, of 100 m (transverse to flow) and

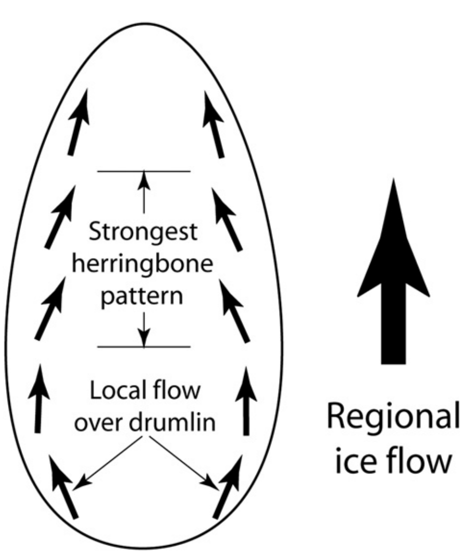

Figure. 2. Herringbone pattern of ice flow over a drumlin. This pattern is predicted theoretically and verified by some till-fabric analyses.

amplitude, h_o, of 5 m. The wavelength is based on a study of drumlin fields of the United Kingdom by Clark et al. (2009), in which they found that the smallest were ~200 m long and ~100 m wide. The choice of an amplitude is not particularly important; any value > 0 will result in topographic focusing of geothermal heat in troughs, and the effect increases linearly with relief.

Turcotte and Schubert start with the temperature distribution in a cross section of a semi-infinite half space, Earth's crust. Consider an orthogonal coordinate system in which the *x*- and *y*-axes are, respectively, parallel and transverse to the long axis of the drumlin, and the *z*-axis is normal to the mean topographic surface and directed downward. The origin is directly beneath the crest of the drumlin and on the mean topographic surface (Fig. 4). The governing equation, is the two-dimensional Laplace equation:

$$\frac{\partial^2\theta}{\partial y^2} + \frac{\partial^2\theta}{\partial z^2} = 0. \tag{1}$$

in which $\theta(y,z)$ is the temperature. Turcotte and Schubert first consider a boundary that is planar rather than sinusoidal, and impose a Dirichlet (temperature) boundary condition:

$$\theta_s(y) = \theta_o + \Delta\theta\cos\frac{2\pi y}{\lambda} \tag{2}$$

where θ_o is the mean temperature along the surface, and $\Delta\theta$ and λ are the amplitude and wavelength of the temperature variation, respectively. Their solution to this problem is:

$$\theta(y,z) = \theta_o + \Delta\theta\cos\frac{2\pi y}{\lambda}\left(c_1 e^{-\frac{2\pi}{\lambda}z} + c_2 e^{\frac{2\pi}{\lambda}z}\right) \tag{3}$$

where c_1 and c_2 are constants to be determined from the boundary conditions. To ensure that $\theta(y,z)$ remains finite as z becomes large, c_2 must be 0. Then the condition $\theta(y,0) = \theta_s(y)$ requires that $c_1 = 1$.

Turcotte and Schubert next consider the one-dimensional problem of the temperature in a column normal to Earth's surface and extending downward. In the absence of radioactive decay, the temperature in the column is given by:

$$\theta(y,z) = \theta_s(y) + \frac{G}{K}Z \tag{4}$$

where G is the geothermal heat flux [which we take to be 60 mW m^{-2}, the global mean value (Chapman and Pollack, 1975)] and K is the

Please cite this article as: Hooke, R.L., Medford, A., Are drumlins a product of a thermo-mechanical instability?, Quaternary Research (2013), http://dx.doi.org/10.1016/j.yqres.2012.12.002

thermal conductivity ($\mathrm{Js^{-1}\ m^{-2}(°C/m)^{-1}}$). (Dimensionally, G/K is a temperature gradient, °C/m.)

It turns out (below) that $\Delta\theta$ is small (-0.10°C beneath the drumlin crest and -0.20°C beneath the trough) compared with the range of Gz/K over a depth equal to the wavelength of the drumlin (≈3°C). Thus, $\theta_s \cong \theta_o$ (a constant) and (4) is an approximate solution to (1). By the principle of superposition, the sum of Eqs. (3) and (4):

$$\theta(y,z) = \theta_o + \frac{G}{K}z + \Delta\theta\left(\cos\frac{2\pi y}{\lambda}\right)e^{-\frac{2\pi}{\lambda}z} \tag{5}$$

is then also a solution to (1). {However, $\Delta\theta\cos\frac{2\pi y}{\lambda}$ [Eq. (2)] is comparable to $\Delta\theta\cos\frac{2\pi y}{\lambda}e^{-\frac{2\pi}{\lambda}z}$ [Eq. (3)] so neglecting the variation in θ_s with y may not be justified.}

Eq. (5) gives the temperature distribution beneath a *planar* surface across which, in one direction (in our case, normal to ice flow), there is a sinusoidal variation in temperature. We want temperatures beneath a *sinusoidal* topographic surface. Thus, we need to express temperatures on the planar surface in terms of the interface temperature, $\theta_i(y)$, along the sinusoidal surface. Because field evidence suggests that drumlins form with water present, as noted above, we assume the interface temperature in the troughs is the pressure melting temperature, θ_p, which is controlled by the thickness of the overlying ice. On the ridge, on the other hand, the base is slightly below the pressure melting temperature. Let us assume that the frozen layer beneath the ridge is 5% of the ridge height, or 0.25 m beneath the crest, diminishing to 0 at $y = \pm\lambda/4$, or ± 25 m from the crest. Analytically, we represent these assumptions by:

$$\tilde{h} = 0.95h \qquad -\lambda/4 < y < \lambda/4$$
$$\tilde{h} = h \qquad y < -\lambda/4,\ y > \lambda/4$$

where $\tilde{h}(y)$ and $h(y)$ are the height of the pressure melting isotherm and the topographic surface, respectively, above (or below) the mean elevation of the topography. The interface temperature is now:

$$\theta_i = \theta_p = \overline{\theta}_p + \frac{d\theta}{dP}h_o\cos\frac{2\pi y}{\lambda} \qquad \text{Trough} \tag{6a}$$

$$\theta_i = \overline{\theta}_p + \left(0.95\frac{d\theta}{dP} - 0.05\frac{d\theta}{dz}\Big|_i\right)h_o\cos\frac{2\pi y}{\lambda} \qquad \text{Ridges} \tag{6b}$$

Here $\overline{\theta}_p$ [$= \theta_o$ in Eq. (5)] is the pressure melting temperature on $z = 0$, $\frac{d\theta}{dP}$ is change in pressure melting temperature with pressure (-6.6×10^{-4} K/m), and $\frac{d\theta}{dz}\big|_i$ is the mean temperature gradient at the interface. As we are interested only in deviations from the mean temperature, we take $\overline{\theta}_p$ to be 0°C. The resulting temperatures along the surface of our incipient drumlin are shown on the left side of Figure 4. The pressure melting temperature in the bottom of the trough, 5 m below the mean elevation, is -0.0033°C, while that at the base of the frozen layer beneath the crest, 4.75 m above the mean, is $+0.0031$°C. Extrapolating the latter through the frozen layer gives an interface temperature of -0.0019°C.

This temperature variation must now be projected onto the plane. Departing somewhat from Turcotte and Schubert, we set $z = \tilde{h}$ in Eq. (5) to obtain the pressure melting temperature, θ_p, thus:

$$\theta_p\left(y,\tilde{h}\right) = \frac{G}{K}\tilde{h} + \Delta\theta\left(\cos\frac{2\pi y}{\lambda}\right)e^{-\frac{2\pi}{\lambda}\tilde{h}} \tag{7}$$

Of these quantities, all are known except $\Delta\theta$, for which we may thus solve:

$$\Delta\theta = \frac{\theta_p - \frac{G}{K}\tilde{h}}{\left(\cos\frac{2\pi y}{\lambda}\right)e^{-\frac{2\pi}{\lambda}\tilde{h}}} \tag{8}$$

Eq. (5) now becomes:

$$\theta(y,z) = \frac{G}{K}z + \left(\theta_p - \frac{G}{K}\tilde{h}\right)e^{-\frac{2\pi}{\lambda}(z-\tilde{h})} \tag{9}$$

This is our solution for the temperature distribution at depth beneath the drumlin. It differs from the Turcotte and Schubert solution only in the presence of $\tilde{h}$. The presence of $\tilde{h}$ in the exponential term means that on $z = \tilde{h}$ our solution gives $\theta(y,\tilde{h}) = \theta_p$, as expected, whereas the Turcotte and Schubert solution does not.

The drumlin profile and the distribution of temperature with depth at two points, 5 m from the drumlin crest and 10 m short of the adjacent trough, are shown in Figure 5. Note that a *steeper* line on these plots represents a *lower* temperature gradient, $d\theta/dz$, and thus a *lower* heat flux. By comparing the temperature distributions with the mean geothermal gradient (light dashed lines) one can see that the gradient beneath the drumlin crest is less than the mean and that beneath the trough is greater. This means that more of the geothermal heat will be focused on the trough and less on the crest.

Loss of heat upward into the ice

Two possible fates await this heat: (*i*) it may be consumed by melting basal ice, or (*ii*) it may be conducted upward into the ice. In our model, in which there is a frozen layer beneath the ridge, all of the heat reaching the interface within $\pm\lambda/4$ of the perturbation crest is conducted upward into the ice.

If the ice were not moving, it would be a simple matter to calculate this upward heat flux. The problem differs from that which we just solved only in that now the term containing $e^{-\frac{2\pi}{\lambda}z}$ in Eq. (3) must be discarded so that $\theta(y,z)$ remains finite at large negative z, and the term

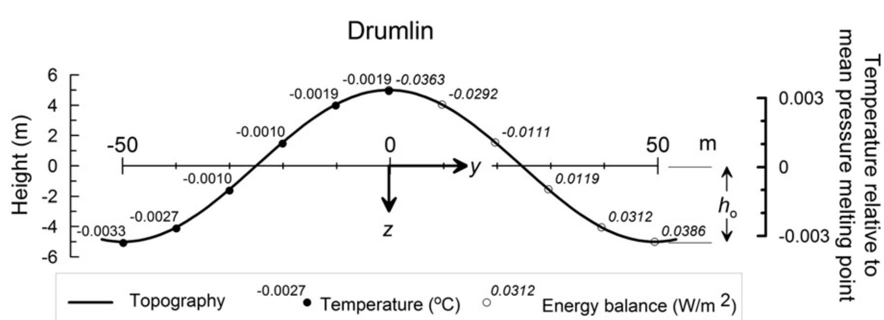

Figure. 4. Drumlin profile and coordinate system used in calculations. Temperatures (°C) along ice/bed interface are shown to left of the crest (see text for explanation), and energy balances ($\mathrm{W/m^2}$) to right. A negative energy balance implies freezing.

Please cite this article as: Hooke, R.L., Medford, A., Are drumlins a product of a thermo-mechanical instability?, Quaternary Research (2013), http://dx.doi.org/10.1016/j.yqres.2012.12.002

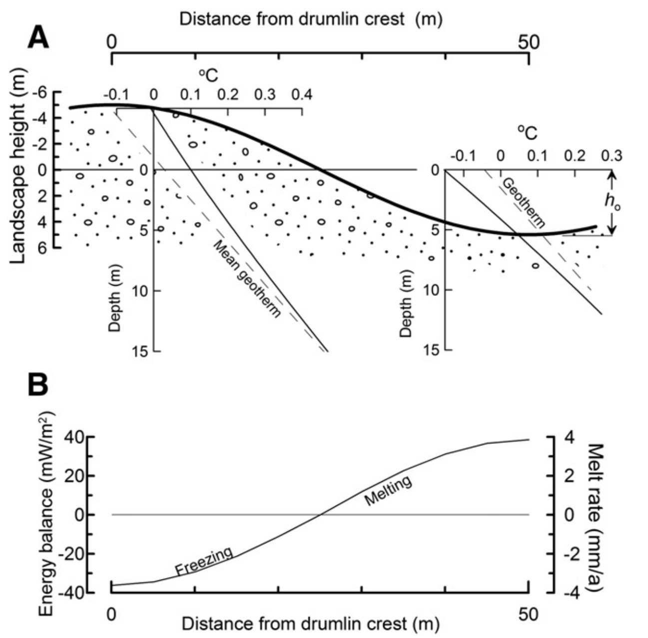

Figure. 5. A. Temperature profiles beneath the crest of a drumlin and beneath the adjacent trough, calculated using $G = 60$ mW/m^2 and $K = 2.09$ W/mK for both substrate and ice. See text for discussion. **B**. Calculated energy balance and melt rate at interface.

involving $e^{+\frac{2\pi}{\lambda}z}$ must be retained. Thus, the solution becomes:

$$\theta(y,z) = \frac{G}{K}z + \left(\theta_i - \frac{G}{K}h\right)e^{\frac{2\pi}{\lambda}(z+h)} \qquad (10)$$

This is a sort of a mirror image of the solution in Figure 5, with heat conduction upward into the ice away from the drumlin trough exceeding that conducted away from the crest (Fig. 3).

Solution

The combined solution for the substrate and the stationary ice is illustrated in Figure 6. Steeper lines on this plot now represent higher heat fluxes. Thus, in the trough, more heat reaches the interface from below than is conducted upward into the ice, and basal ice would melt. Beneath the crest, the reverse is true and any water arriving there would refreeze. The energy balance for these conditions is shown in Figure 5B. In the steady-state, this solution yields ~4 mm/a of basal ice melt in the trough and ~4 mm/a of refreezing on the drumlin crest. With a till porosity of ~15% (Fetter, 1994), the growth rate at the crest of our 5 m amplitude perturbation would be ~25 mm/a. If the amount of material eroded from the trough equaled the amount that could be refrozen at the crest, the increase in relief of the drumlin would be twice this. The growth rate increases linearly with h_o. Thus, a 1 m initial perturbation might take a few hundred years to develop into a typical drumlin, a few tens of meters high. Such a time scale seems reasonable, and is broadly consistent with an estimate by Rose (1989). A drumlin shaped largely by remolding a pre-existing bump on the bed could be formed much more quickly, of course.

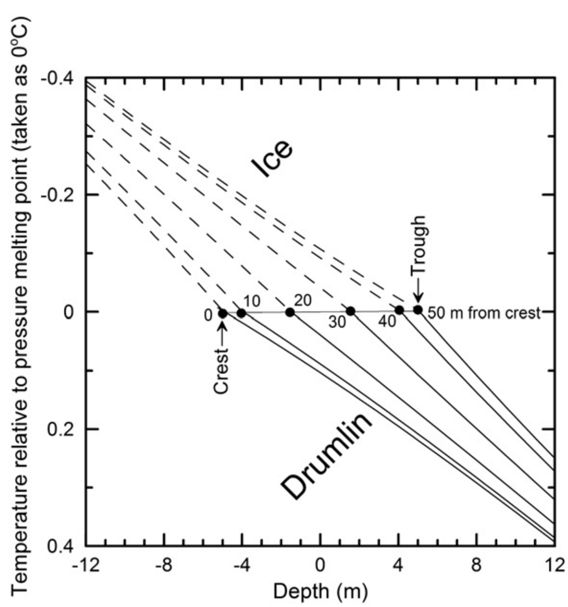

Figure. 6. Variation in temperature with depth in the substrate below the drumlin (solid lines), and with height in the ice above it (dashed lines). Distance from crest is shown. See text for discussion.

Discussion

Because the ice is moving three additional processes must be considered:

(i) Will increased frictional heating in ice over the frozen bed, and thus restrained by it, outweigh effects of the heat flow perturbation?
(ii) How will longitudinal gradients in temperature or other glaciological parameters affect our solution?
(iii) How long will it take the temperature field to adjust to the growing perturbation in the bed as the ice moves over it?

With regard to the first issue, the steady-state temperature gradient in basal ice over a completely frozen bed is adjusted to conduct the combined frictional and geothermal heat upward into the ice. The spatially averaged frictional energy produced is approximately $\tau_b\bar{u}$, where τ_b is the basal traction and $\bar{u}$ is the depth-averaged velocity (Budd, 1969 or Hooke, 2005, p. 132). Beneath an ice sheet 1000 m thick with a surface slope of 0.008 moving over a partially thawed bed, $\tau_b \approx 0.07$ MPa and $\bar{u} \approx 10$ m/a (Hughes, 1992; Hooke, 2005, Fig. 7.29). Thus, the spatially averaged frictional heat flux is ~12 mW/m^2, or about 1/5th of the global mean geothermal flux. These τ_b and $\bar{u}$ conditions are roughly appropriate for a location ~60 km from an ice margin. The negative energy balance at the crest of a developing drumlin with an amplitude of 5 m is >3 times this spatially averaged frictional heat flux (Fig. 5B).

Upon encountering a partially thawed area, sliding is initiated in wet areas separating frozen patches. As the thawed areas increased in size down-flow, frictional heating in the frozen areas likely increases while that in the thawed areas decreases. At the same time, however, the drumlins are likely growing, increasing the negative energy balance over their crests.

In short, under some conditions enhanced frictional energy dissipation over a frozen patch may prevent initiation of a drumlin. However, given that the concentration of friction over the incipient drumlin develops slowly as the extent of the thawed area increases, and that once the drumlin reaches a modest height, the frictional energy released is likely smaller than the negative energy balance at the drumlin crest, we think that many perturbations would be able to grow into full-fledged drumlins.

An approximate solution to the second issue is provided by the Column Model of Budd (1969) [see also Budd et al., 1971, p. 77 or Hooke, 2005, p. 133, Equation (6.40)]. The Column Model provides an estimate of the temperature distribution, θ(z), in a column extending from the surface to the bed of an ice sheet. Without going into detail, the derivative of this temperature with respect to z, evaluated at the bed is:

$$\left.\frac{\partial\theta}{\partial z}\right|_{z=0} = \frac{1}{K\sqrt{\frac{w_{surf}}{2H\kappa}}}\left(G + \tau_b\bar{u}\right) \qquad (11)$$

Please cite this article as: Hooke, R.L., Medford, A., Are drumlins a product of a thermo-mechanical instability?, Quaternary Research (2013), http://dx.doi.org/10.1016/j.yqres.2012.12.002

in which w_{surf} is the vertical ice velocity at the glacier surface, H is the glacier thickness, and κ is the thermal diffusivity of ice. $\frac{\partial\theta}{\partial z}|_{z=0}$ is a gradient sufficient to conduct upward the mean combined geothermal and frictional heat. It does not take into consideration variations in the heat flux to the interface from the substrate due to variations in K or topographic focusing. Because G, τ_b, $\overline{u}$, w_{surf}, and H are likely to change only slowly in the longitudinal direction over distances of the scale of drumlin length, $\frac{\partial\theta}{\partial z}|_{z=0}$ is likely nearly constant. Thus, we need not be overly concerned by the possibility that basal-ice temperature gradients advected from upglacier will invalidate our argument.

Regarding the third question, consider ice flowing across a planar substrate toward a 'drumlin' consisting of a rectangular block of till, a few meters high, with a half-width of 50 m and a length several times that. Some distance down-flow from the up-ice end of this block, beyond the influence of boundary effects, the ice temperature in a direction transverse to the flow is no longer uniform. There is now a temperature gradient from the top of the block to the adjacent bed, driving heat toward the lower temperature at the drumlin crest. The problem is similar to one in which a block of half-width $L = 50$ m (y-direction), infinite length (x-direction), and uniform positive temperature $\delta\theta$ is plunged into an ice bath at 0°C, and the temperature in the block gradually decays toward 0°C. The temperature in the block as a function of distance from the edge, y, and time, t, is given by:

$$\theta(y,t) = \frac{4}{\pi}\delta\theta \sum \frac{1}{m}\left(\sin\frac{m\pi y}{2L}\right)e^{\frac{-m^2\pi^2\kappa t}{4L^2}} \qquad m = 1, 3, 5, \ldots \tag{12}$$

(Nearing, 2012). After 25 yr the temperature in the center of the block, $\theta(50, 25)$ is ~0.5 $\delta\theta$. In other words, the temperature will have progressed only about halfway to its new equilibrium value. If $\overline{u}$ is modest — say 10 m/a, as above — the ice, by this time, will have moved about 250 m down the block.

The upshot of these calculations is that although the temperature in the basal ice and the developing drumlin will be continually adjusting, the negative energy balance at the drumlin crest and the positive balance in the trough should persist, so perturbations should grow.

This provides a segue to two interesting questions: (*i*) what limits drumlin length and height, and (*ii*) why, in some areas, do drumlins appear to give way to flutes in the down-ice direction? We offer speculative answers to both. It has been shown that small increases in water pressure can cause an increase in the rate of deformation of subglacial till, but larger increases decouple the glacier from the bed and the rate of till deformation decreases (Hooke et al., 1997). Our model suggests that melting in troughs should increase both down-flow and with time as the temperature distribution adjusts, the phase-change boundary retreats, and fraction of the bed that is frozen decreases, and should also increase as the drumlin grows higher. If the increase in melting reduces the coupling between the ice and the till, it may reduce and eventually stop sediment transfer onto a developing drumlin, limiting its height.

These changes would increase the sliding speed. This might: (*i*) streamline existing drumlins, (*ii*) increase the ability of the glacier to drag till while at the same time reducing the convergence of the flow with the drumlin axis, thus favoring formation of flutes (see also discussion in Schoof and Clarke, 2008, esp. paragraph 45) (*iii*) reorient fabrics on drumlin flanks, erasing earlier herringbone patterns. These speculations underscore our lack of understanding of the conditions under which glaciers move significant amounts of till by basal drag.

Conclusions

We hypothesize that a common mode of drumlin formation by continental ice sheets involves a positive feedback process that operates in a zone over which a cold bed up-ice transitions to a thawed bed down-ice. Cold patches surrounded by thawed bed are present in this zone. These may be on low hills or over areas of lower substrate thermal conductivity. Drag provided by the cold patches deflects ice flow diagonally into their lees (Fig. 2). Wet till dragged by this flow refreezes, increasing the height of the frozen patch and deepening the adjacent thawed zone. The developing topography deflects geothermal heat toward the lower areas, setting up a positive feedback process leading to growth of a drumlin.

This process only operates in zones over which a frozen bed transitions to a thawed one, or conversely. Alternation of frozen and thawed patches in such zones is undoubtedly common. Transitions from a frozen to a thawed bed are likely in the zone beneath the equilibrium line. In this respect, our proposed mechanism is consistent with the observation that where drumlins and end moraines can be related, the drumlins are commonly found a few tens of kilometers up-ice from associated end moraines. We note also that drumlins did not form beneath the interior of the Scandinavian ice sheet in places where cold patches survived through the last glaciation (Kleman and Borgström, 1994), suggesting that conditions near the margin, perhaps in particular, a compressive flow regime, may be a prerequisite for their formation. Our model is also consistent with the observation that drumlins may be either depositional or erosional, and in the extreme may be composed entirely of bedrock (rock drumlins).

Acknowledgments

Our contribution herein has been to assemble diverse ideas from a number of sources into what we think is a coherent conceptual model for drumlin formation. We salute those who have gone before us: those whose ideas have stimulated our thinking and especially those whose careful observations in the field have constrained the characteristics of drumlins and the conditions under which they formed. C. Clark called our attention to his 2010 paper which led us to several other references on drumlin formation. We appreciate B. Hanson's clarification of certain aspects of the calculations.

References

Andrews, J.T., King, C.A.M., 1968. Comparative till fabrics and till fabric variability in a till sheet and a drumlin: a small scale study. Proceedings of the Yorkshire Geological Society 36 (4), 435–461.

Armstrong, J.E., Tipper, H.W., 1948. Glaciation in north central British Columbia. American Journal of Science 246, 283–310.

Baranowski, S., 1977. Regularity of drumlin distribution and the origin of their formation. Studia Studia Geologica Polonica 52, 53–68.

Bell, R.E., et al., 2011. Widespread persistent thickening of the East Antarctic Ice Sheet by freezing from the base. Science 331, 1592–1595.

Benn, D.I., 1994. Fluted moraine formation and till genesis below a temperate valley glacier: Slettmarkbreen, Jotunheiman, southern Norway. Sedimentology 41 (2), 279–292.

Budd, W.F., 1969. The dynamics of ice masses. Australian National Antarctic Expeditions Scientific Reports, Series A (IV) Glaciology. (Publication No. 108).

Budd, W.F., Jensen, D., Radok, U., 1971. Derived physical characteristics of the Antarctic Ice Sheet. *Australian National Antarctic Expeditions Interim Reports*, Series A (IV) Glaciology. (Publication No. 120).

Chapman, D.S., Pollack, H.N., 1975. Global heat flow: a new look. Earth and Planetary Science Letters 28, 23–32.

Chapwanya, M., Clark, C.D., Fowler, A.C., 2011. Numerical computations of a theoretical model of ribbed moraine formation. Earth Surface Processes and Landforms 36, 1105–1112.

Clark, C.D., 1999. Glaciodynamic context of subglacial bedform generation and preservation. Annals of Glaciology 28, 23–32.

Clark, C.D., 2010. Emergent drumlins and their clones: from till dilatancy to flow instabilities. Journal of Glaciology 51 (200), 1011–1025.

Clark, C.D., Hughes, A.L.C., Greenwood, S.L., Spagnolo, M., Ng, F.S.L., 2009. Size and shape characteristics of drumlins, derived from a large sample, and associated scaling laws. Quaternary Science Reviews 28, 677–692.

Colgan, P., Mickelson, D.M., 1997. Genesis of streamlined landforms and flow history of the Green Bay Lobe, Wisconsin, USA. Sedimentary Geology 111, 7–25.

Embleton, C., King, C.A.M., 1975. Glacial Geomorphology. John Wiley, New York . (573pp.).

Fetter, C.W., 1994. Applied Hydrology, 3rd ed. McMillian Publishing Co., New York . (691 pp.).

Fowler, A.C., 2000. An instability Mechanism for Drumlin Formation. In: Maltman, A., Hambrey, M.J., Hubbard, B. (Eds.), Deformation of Subglacial Materials. : Special

Publications Geological Society, 176. The Geological Society, London, UK, pp. 307–319.
Fowler, A.C., 2009. Instability modelling of drumlin formation incorporating lee-side cavity growth. Proceedings of the Royal Society, London, Series A 465, 2681–2702.
Fowler, A.C., 2010. The instability theory of drumlin formation applied to Newtonian viscous ice of finite depth. Proceedings of the Royal Society, London, Series A 466 (2121), 2673–2694.
Goldstein, B., 1994. Drumlins of the Puget Lowland, Washington state, USA. Sedimentary Geology 91, 299–311.
Hart, J.K., 1995. Recent drumlins, flutes and lineations at Vestari-Hagafellsjökull, Iceland. Journal of Glaciology 41 (139), 596–606.
Hättestrand, C., Kleman, J., 1999. Ribbed moraine formation. Quaternary Science Reviews 18, 43–61.
Hess, D.P., Briner, J.P., 2009. Geospatial analysis of controls on subglacial bedform morphometry in the New York Drumlin Field — implications for Laurentide Ice Sheet dynamics. Earth Surface Processes and Landforms 34, 1126–1135.
Hindmarsh, R.C.A., 1998. Drumlinization and drumlin-forming instabilities: viscous till mechanisms. Journal of Glaciology 44 (147), 293–314.
Hindmarsh, R.C.A., 1999. Coupled ice–till dynamics and the seeding of drumlins and bedrock forms. Journal of Glaciology 28, 221–230.
Hooke, R. LeB., 1977. Basal temperatures in polar ice sheets: a qualitative review. Quaternary Research 7, 1–13.
Hooke, R. LeB., 2005. Principles of Glacier Mechanics. Cambridge University Press . (429 pp.).
Hooke, R. LeB., Hanson, B., Iverson, N.R., Jansson, P., Fischer, U.H., 1997. Rheology of till beneath Storglaciären, Sweden. Journal of Glaciology 43 (143), 172–179.
Hughes, T., 1992. Abrupt climate change related to unstable ice-sheet dynamics: toward a new paradigm. Palaeogeography, Palaeoclimatology, Palaeoecology 97, 203–234.
Jansson, K.N., Kleman, J., 1999. The horned craig-and-tails of the Ungava Bay landform swarm, Quebec-Labrador, Canada. Annals of Glaciology 28, 168–174.
Johnson, M.D., Schomacker, A., Benediktsson, Í.Ö., Geiger, A.J., Ferguson, A., Ingólfson, Ó., 2010. Active drumlin field revealed at the margin of Múlajökull, Iceland: a surge type glacier. Geology 38 (10), 943–946.
Kleman, J., Borgström, I., 1994. Glacial landforms indicative of a partly frozen bed. Journal of Glaciology 40 (135), 255–264.
Kleman, J., Glasser, N.F., 2007. The subglacial thermal organisation (STO) of ice sheets. Quaternary Science Reviews 26, 585–597.
Lachenbruch, A.H., 1968. Rapid estimation of the topographic disturbance to superficial thermal gradients. Reviews of Geophysics 6 (3), 365–400.
Lees, C.H., 1910. On the shapes of the isogeotherms under the mountain ranges in radio-active districts. Proceedings of the Royal Society of London, Series A 83 (563), 339–346.
Menzies, J., Zaniewski, K., Dreger, D., 1997. Evidence, from microstructures, of deformable bed conditions within drumlins, Chimney Bluffs, New York State. Sedimentary Geology 111, 161–175.
Nearing, J., 2012. Partial differential equations. www.Physics.miami.edu/~nearing/mathmethods.pde.pdf (Last accessed April 2012).
Oswald, G.K.A., Gogineni, S.P., 2008. Recovery of subglacial water extent from Greenland radar survey data. Journal of Glaciology 54 (184), 94–107.
Patterson, C.J., Hooke, R.L., 1995. Physical environment of drumlin formation. Journal of Glaciology 41 (137), 30–38.
Rose, J., 1989. Glacier stress patterns and sediment transfer associated with the formation of superimposed flutes. Sedimentary Geology 62, 151–176.
Schoof, C., 2007. Pressure-dependent viscosity and interfacial instability in coupled ice–sediment flow. Journal of Fluid Mechanics 570, 227–252.
Schoof, C.G., Clarke, G.K.C., 2008. A model for spiral flows in basal ice and the formation of subglacial flutes based on a Reiner–Rivlin rheology for glacial ice. Journal of Geophysical Research 113 (B5) http://dx.doi.org/10.1029/2007JB004957.
Shaw, J., Freschauf, R.C., 1973. A kinematic discussion of the formation of glacial flutings. The Canadian Geographer 17 (1), 19–35.
Smalley, I.J., Unwin, D.J., 1968. The formation and shape of drumlins and their distribution and orientation in drumlin fields. Journal of Glaciology 7 (51), 377–390.
Smith, A.M., Murray, T., Nicholls, K.W., Makinson, K., Aðalgeirsdóttir, G., Behar, A.E., Vaughn, D.G., 2007. Rapid erosion, drumlin formation, and changing hydrology beneath an Antarctic ice stream. Geology 35 (2), 127–130.
Stanford, S.D., Mickelson, D.M., 1985. Till fabric and deformational structures in drumlins near Waukesha, Wisconsin, U.S.A. Journal of Glaciology 31 (109), 220–228.
Stokes, C.R., Spagnolo, M., Clark, C.D., 2011. The composition and internal structure of drumlins: complexity, commonality, and implications for a unifying theory of their formation. Earth-Science Reviews 107, 398–422.
Tarr, R.S., 1894. The origin of drumlins. The American Geologist 13, 393–407.
Turcotte, D.L., Schubert, G., 2002. Geodynamics: Applications of Continuum Physics to Geological Problems, 2nd ed. John Wiley & Sons, Inc., U.S.A . (464 pp.).
Walker, M.J.C., 1973. The nature and origin of a series of elongated ridges in the Morley Flats area of the Bow Valley, Alberta. Canadian Journal of Earth Sciences 10, 1340–1346.

Please cite this article as: Hooke, R.L., Medford, A., Are drumlins a product of a thermo-mechanical instability?, Quaternary Research (2013), http://dx.doi.org/10.1016/j.yqres.2012.12.002